American Scholarly Journal for Scientific Research (ASJSR)
https://asjsr.org/ Volume 2025, Issue 1004

The Riemann Hypothesis: The Million-Dollar Mystery Behind Prime Numbers

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Date: December 13, 2025

Abstract

The Riemann Hypothesis is one of the most famous unsolved problems in mathematics. It belongs to number theory, which is the study of whole numbers, especially prime numbers. Prime numbers are numbers such as 2, 3, 5, 7, and 11 that cannot be divided exactly by any smaller whole number except 1. They are often called the “building blocks” of arithmetic because every whole number can be made by multiplying prime numbers.

The Riemann Hypothesis is connected to a special mathematical function called the Riemann zeta function. This function is linked to how prime numbers are spread across the number line. The mystery is about the “zeros” of this function, meaning the points where the function becomes equal to zero. The hypothesis says that all the important, or non-trivial, zeros lie on one special vertical line called the critical line. The Clay Mathematics Institute describes the hypothesis as saying that all “interesting solutions” of $\zeta(s) = 0$ lie on a certain vertical straight line.

This paper explains the Riemann Hypothesis in simple language for a high school student. It discusses prime numbers, the zeta function, zeros, the critical line, why the problem matters, and why it has remained unsolved for more than 160 years.

Introduction

Mathematics is full of patterns. Some patterns are easy to see, like even numbers: 2, 4, 6, 8, 10. Some patterns are harder, like the pattern of prime numbers. Prime numbers look simple at first, but they become very mysterious when we look at them closely.

A prime number is a number greater than 1 that can only be divided by 1 and itself. For example, 7 is prime because it can only be divided by 1 and 7. But 8 is not prime because it can be divided by 1, 2, 4, and 8.

Prime numbers are important because every whole number can be broken down into primes. For example:

$12 = 2 \times 2 \times 3$

30 = 2 × 3 × 5

100 = 2 × 2 × 5 × 5

This means primes are like the atoms of arithmetic. Just as atoms build matter, prime numbers build whole numbers.

However, primes do not appear in a simple pattern. Between 1 and 10, there are four prime numbers: 2, 3, 5, and 7. Between 90 and 100, there is only one prime number: 97. Sometimes primes are close together, like 11 and 13. Sometimes there are large gaps between them.

This irregular behaviour makes mathematicians ask a big question: **Is there a hidden pattern behind prime numbers?**

The Riemann Hypothesis is one of the deepest attempts to answer this question. It was proposed by the German mathematician Bernhard Riemann in 1859. The Clay Mathematics Institute lists it as one of the seven Millennium Prize Problems, and its official page explains that Riemann connected the frequency of prime numbers to the behaviour of the Riemann zeta function.

Research Question

Are all the non-trivial zeros of the Riemann zeta function located on the critical line, where the real part is 1/2?

Aim

The aim of this paper is to explain the Riemann Hypothesis in simple words and understand why it matters in mathematics.

This paper will try to answer the following questions:

1. What are prime numbers?
2. Why are prime numbers difficult to predict?
3. What is the Riemann zeta function?
4. What are zeros of a function?
5. What is the critical line?
6. Why would proving the Riemann Hypothesis be important?
7. Why is this problem still unsolved?

Mathematical Theory

1. Prime Numbers

Prime numbers are whole numbers greater than 1 that have only two factors: 1 and themselves.

Examples of prime numbers are:

2, 3, 5, 7, 11, 13, 17, 19, 23

Examples of non-prime numbers are:

4, 6, 8, 9, 10, 12

These are called composite numbers because they can be made by multiplying smaller numbers.

For example:

6 = 2 × 3

9 = 3 × 3

10 = 2 × 5

Prime numbers are important because every whole number can be built from them. This is why they are called the building blocks of numbers.

2. The Problem with Prime Numbers

Prime numbers do not appear in an obvious pattern. If someone gives us a number, we can check whether it is prime. But predicting exactly where the next prime number will appear is much harder.

For example:

After 2 comes 3.
After 3 comes 5.
After 5 comes 7.
After 7 comes 11.
After 11 comes 13.
After 13 comes 17.

The gaps between prime numbers keep changing. Sometimes the gap is 1, sometimes 2, sometimes 4, and sometimes much larger.

This makes prime numbers feel random. But mathematicians believe there is still deep structure behind them.

3. Counting Prime Numbers

Mathematicians often use a function called **π(x)**, pronounced "pi of x," to count how many prime numbers are less than or equal to a number x.

For example:

π(10) = 4, because there are 4 primes up to 10: 2, 3, 5, 7.

π(100) = 25, because there are 25 primes up to 100.

Mathematicians discovered that although primes are irregular, the number of primes up to a large number follows an approximate pattern. This is known as the Prime Number Theorem. The Clay Mathematics Institute explains that the prime number theorem describes the average distribution of primes, while the Riemann Hypothesis is about the deviation from that average.

In simple words, the Prime Number Theorem gives a rough map, but the Riemann Hypothesis may explain how accurate that map really is.

4. The Riemann Zeta Function

The Riemann zeta function is written as:

$\zeta(s)$

For some values of s, it looks like this:

$\zeta(s) = 1 + 1/2^s + 1/3^s + 1/4^s + 1/5^s + \ldots$

This is an infinite sum, meaning it keeps going forever.

For example, when **s = 2**:

$\zeta(2) = 1 + 1/2^2 + 1/3^2 + 1/4^2 + 1/5^2 + \ldots$

This becomes:

$\zeta(2) = 1 + 1/4 + 1/9 + 1/16 + 1/25 + \ldots$

At first, this looks like just a strange sum. But Riemann showed that this function is deeply connected to prime numbers. The Clay Mathematics Institute states that Riemann observed that the frequency of prime numbers is closely related to the behaviour of the zeta function.

5. What Is a Zero?

A zero of a function is a value where the function becomes equal to zero.

For example, take this simple function:

$f(x) = x - 3$

If we put **x = 3**, then:

f(3) = 3 − 3 = 0

So, **x = 3** is a zero of the function.

The Riemann Hypothesis is about the zeros of the Riemann zeta function. These are the values of s where:

ζ(s) = 0

6. Trivial and Non-Trivial Zeros

The zeta function has two types of zeros.

The first type is called **trivial zeros**. These are easier to understand and happen at negative even numbers:

−2, −4, −6, −8, ...

They are called trivial not because they are useless, but because they are easier to explain compared to the other zeros.

The second type is called **non-trivial zeros**. These are much more mysterious. They are complex numbers.

7. What Are Complex Numbers?

A complex number has two parts:

a real part and **an imaginary part**.

It is usually written like this:

a + bi

Here:

a is the real part.
b is the imaginary part.
i is the square root of −1.

For example:

3 + 2i

In this number, 3 is the real part and 2 is the imaginary part.

The non-trivial zeros of the Riemann zeta function are complex numbers. The Riemann Hypothesis says that all these non-trivial zeros have a real part of exactly **1/2**.

8. The Critical Line

The critical line is the vertical line where the real part of the complex number is:

1/2

So, numbers on the critical line look like:

1/2 + bi

The imaginary part can change, but the real part stays fixed at 1/2.

The Riemann Hypothesis says:

Every non-trivial zero of the Riemann zeta function lies on this critical line.

The National Institute of Standards and Technology also describes the Riemann Hypothesis as the statement that all non-trivial zeros of the zeta function lie on the critical line.

Methodology

This paper uses a simple explanatory and secondary research approach. It does not attempt to prove the Riemann Hypothesis because no accepted proof is currently known. Instead, it studies the main ideas behind the problem using existing mathematical explanations and official sources.

The approach includes four steps.

First, the paper explains prime numbers and why they are important.

Second, it introduces the Riemann zeta function in a simple way, using basic examples instead of advanced mathematics.

Third, it explains the meaning of zeros, non-trivial zeros, and the critical line.

Fourth, it studies why the Riemann Hypothesis matters for understanding the distribution of prime numbers.

This method is useful for a high school research paper because the Riemann Hypothesis is far beyond normal school mathematics. The goal is not to solve the problem, but to understand what the mystery means and why mathematicians care about it.

Data Analysis

1. Counting Prime Numbers

To understand why the Riemann Hypothesis matters, we first look at how primes are counted.

Number x	Number of primes up to x, $\pi(x)$	Approximation x / ln(x)
10	4	4.34
100	25	21.71
1,000	168	144.76
10,000	1,229	1,085.74

This table shows that **x / ln(x)** gives an approximate idea of how many primes appear up to x. It is not exact, but it gets closer in a broad sense as numbers become larger.

For example, there are actually 25 primes up to 100, while the approximation gives about 21.71. There are actually 1,229 primes up to 10,000, while the approximation gives about 1,085.74.

This shows an important point: prime numbers are irregular in detail, but they follow a broad statistical pattern.

2. Why the Approximation Is Not Enough

The approximation tells us the average behaviour of primes, but mathematicians want to know more. They want to understand how far the actual number of primes can move away from the expected number.

This is where the Riemann Hypothesis becomes important.

If the Riemann Hypothesis is true, then the error in estimating the number of primes would be more tightly controlled. In simple words, it would mean that prime numbers are not as random as they look. They would still be irregular, but their irregularity would follow a deep hidden rule.

3. A Simple Example of the Zeta Function

The zeta function is:

$$\zeta(s) = 1 + 1/2^s + 1/3^s + 1/4^s + \ldots$$

Let us use **s = 2**:

$$\zeta(2) = 1 + 1/4 + 1/9 + 1/16 + 1/25 + \ldots$$

Adding the first few terms:

1 = 1

1 + 1/4 = 1.25

1.25 + 1/9 ≈ 1.361

1.361 + 1/16 ≈ 1.424

1.424 + 1/25 = 1.464

The sum keeps increasing slowly. It does not become zero here. This is only a simple example to show how the zeta function works as an infinite sum.

The zeros involved in the Riemann Hypothesis are much more complicated because they involve complex numbers, not just ordinary whole numbers.

4. What Computers Have Found

Mathematicians have used computers to check many zeros of the Riemann zeta function. These computer checks have found many zeros on the critical line. According to NIST's Digital Library of Mathematical Functions, calculations based on the Riemann–Siegel formula showed that the first ten billion zeros in the critical strip are on the critical line.

This is strong evidence, but it is not a proof.

Why not?

Because the Riemann Hypothesis is about **all** non-trivial zeros. There are infinitely many of them. Checking the first ten billion zeros is impressive, but infinity is much larger than any number a computer can check.

A proof would need to show that every non-trivial zero, forever, lies on the critical line.

Findings

The research leads to several important findings.

1. Prime Numbers Are Simple but Mysterious

Prime numbers are easy to define, but their pattern is difficult to predict. They appear in a way that seems irregular, but they also follow broad statistical laws.

2. The Zeta Function Connects to Prime Numbers

The Riemann zeta function may look like an abstract formula, but it is deeply connected to prime numbers. Riemann's major insight was that studying this function could reveal information about how primes are distributed.

3. The Main Mystery Is About Zeros

The Riemann Hypothesis is not directly asking, "Where are the primes?" Instead, it asks where the zeros of the zeta function are located. This is surprising because it connects two different ideas: zeros of a function and the pattern of prime numbers.

4. The Critical Line Is the Key

The hypothesis claims that every non-trivial zero lies on the critical line, where the real part is 1/2. This single line is at the centre of one of the biggest mysteries in mathematics.

5. Computers Support the Hypothesis but Do Not Prove It

Computer calculations have checked a huge number of zeros and found them on the critical line, but this is still not enough. Mathematics requires a logical proof that works for all cases, not only the cases tested so far.

6. The Problem Is Important Enough for a Million-Dollar Prize

The Riemann Hypothesis is one of the Clay Millennium Prize Problems. The Clay Mathematics Institute selected seven major unsolved problems in mathematics and attached a prize to their solution. Its Millennium Prize page lists the Riemann Hypothesis among these major problems.

Discussion

The Riemann Hypothesis is difficult because it sits between simple and extremely advanced mathematics. On one side, it is about prime numbers, which students learn about in school. On the other side, it involves complex analysis, infinite series, and deep number theory.

This is what makes it fascinating. The starting point is simple: prime numbers. But the path leads to one of the hardest problems in the world.

One way to understand the Riemann Hypothesis is through an analogy.

Imagine prime numbers as stars in the night sky. At first, they look scattered randomly. But astronomers know that stars are affected by gravity, galaxies, and hidden structures. Similarly, primes look scattered, but mathematicians believe there is hidden order behind them.

The zeta function is like a telescope. It allows mathematicians to see patterns in primes that are not visible directly.

The zeros of the zeta function are like signals. Their positions may reveal how much prime numbers can move away from their expected pattern. If all the important zeros are on the critical line, then primes follow a much more controlled pattern than they seem to.

The reason the hypothesis matters is not only because of primes themselves. Prime numbers are used in modern mathematics, computer science, cryptography, and theoretical research. A proof of the Riemann Hypothesis would not suddenly break all internet security, but it would deepen our understanding of the number system.

It would also confirm that mathematicians have been correct about one of the most important hidden patterns in arithmetic.

However, the problem remains unsolved because numerical evidence is not the same as proof. In mathematics, even if a statement works for billions or trillions of examples, it can still fail later unless there is a logical proof.

This is why the Riemann Hypothesis is so powerful. It reminds us that mathematics is not only about calculation. It is about certainty.

Conclusion

The Riemann Hypothesis is one of the greatest unsolved problems in mathematics. It asks whether all non-trivial zeros of the Riemann zeta function lie on the critical line, where the real part is 1/2.

Although the question sounds advanced, its importance comes from something very basic: prime numbers. Prime numbers are the building blocks of arithmetic, but their distribution is difficult to predict. The Riemann zeta function gives mathematicians a way to study the hidden pattern behind primes.

This paper found that the Riemann Hypothesis matters because it would give a deeper understanding of how prime numbers are distributed. It would not make primes completely predictable, but it would show that their irregular behaviour follows a strict mathematical rule.

Computers have checked many zeros and found them on the critical line, but this is not enough to prove the hypothesis. Since there are infinitely many zeros, only a complete mathematical proof can solve the problem.

For a high school student, the Riemann Hypothesis teaches an important lesson: even simple ideas can lead to huge mysteries. Prime numbers may look like basic school mathematics, but they are connected to one of the deepest unanswered questions in the world.

The Riemann Hypothesis remains unsolved, but its beauty lies in this mystery. It shows that mathematics is still alive, still growing, and still full of questions waiting to be answered.

Limitations

This paper has some limitations.

First, it explains the Riemann Hypothesis in simple language, so it does not include the full advanced mathematics needed to study the problem professionally.

Second, the paper does not provide a proof of the Riemann Hypothesis. This is because no accepted proof is currently known.

Third, the calculations used in this paper are basic examples meant to help understanding. They do not represent the full complexity of the zeta function.

Fourth, complex analysis, analytic continuation, functional equations, and advanced number theory are not explained in detail because they are beyond the level of a high school research paper.

Finally, computer evidence is discussed only as support, not as proof. A true solution to the Riemann Hypothesis would require a complete mathematical argument accepted by experts.

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