
The Navier–Stokes Equations: The Math Behind Flowing Water, Air, Smoke, and Storms

Author: Josh Sen

Institution: Mission San Jose, California

Mentor: Thomas Green

Date: October 25, 2025

Abstract

The Navier–Stokes equations are some of the most important equations in science and mathematics. They describe how fluids move. A fluid is anything that can flow, such as water, air, smoke, steam, oil, blood, or lava. These equations help scientists understand rivers, airplane flight, weather, ocean waves, tornadoes, and even how blood moves through the body.

Even though fluids are part of everyday life, their motion is very difficult to predict. Water can swirl, air can form storms, and smoke can twist into strange shapes. The Navier–Stokes equations try to explain this movement by studying speed, direction, pressure, friction, and outside forces like gravity.

The surprising part is that these equations were written in the 1800s, but mathematicians still do not fully understand them. The Clay Mathematics Institute lists the Navier–Stokes problem as one of the Millennium Prize Problems and says that these equations govern the flow of fluids such as water and air, but basic questions about their solutions remain unproved.

This paper explains the Navier–Stokes equations in extremely simple language for a high school student. It does not try to solve the problem. Instead, it explains what the equations mean, why they matter, and why they are still one of the biggest mysteries in mathematics.

Introduction

Imagine pouring milk into coffee. At first, the milk enters as a white stream. Then it curls, spreads, twists, and slowly mixes with the coffee. Now imagine smoke rising from a candle. It may start as a smooth line, but soon it begins to wiggle and swirl. Or think about a river flowing around rocks. The water does not move in a straight line. It spins, splashes, slows down, speeds up, and forms little whirlpools.

All of these examples involve fluids.

In science, a **fluid** is anything that can flow. This includes liquids like water, juice, oil, and lava. It also includes gases like air, steam, and wind. The Navier–Stokes equations are mathematical rules that describe how these fluids move.

At first, this sounds simple. We may think, “Water just flows. Air just moves.” But fluids are actually very complicated. A single drop of water is affected by gravity, pressure, friction, nearby drops, and the surface it touches. Air in a storm is affected by temperature, pressure, rotation, and wind speed. Because of this, fluid motion can become extremely difficult to predict.

The Navier–Stokes equations are used in science, engineering, weather prediction, medicine, aircraft design, rockets, ships, and computer simulations. NASA explains that these equations describe how the velocity, pressure, temperature, and density of a moving fluid are related.

However, there is still a deep mathematical mystery. Mathematicians do not know whether the equations always behave nicely in three dimensions, or whether they can suddenly produce a “blow-up,” meaning the math becomes uncontrollable or infinite. This is why the problem is worth \$1 million.

Research Question

Do the Navier–Stokes equations always give smooth and reasonable solutions for fluid motion, or can their solutions break down and become impossible to control?

Aim: The aim of this paper is to explain the Navier–Stokes equations in simple language so that even someone who does not enjoy mathematics can understand the problem.

This paper will try to answer these questions:

1. What is a fluid?
2. What do the Navier–Stokes equations describe?
3. Why do fluids behave in such complicated ways?
4. What does it mean for a solution to be “smooth”?
5. What does it mean for the math to “blow up”?
6. Why is the problem still unsolved?
7. Why do these equations matter in real life?

Mathematical Theory

1. What Is a Fluid?

A fluid is anything that can flow.

Examples of fluids include:

- Water

- Air
- Smoke
- Steam
- Oil
- Lava
- Blood
- Milk

Liquids and gases are both fluids because they do not keep a fixed shape. If we pour water into a glass, it takes the shape of the glass. If air enters a balloon, it spreads inside the balloon.

This is different from a solid. A rock does not flow and change shape easily. Water and air do.

2. What Do the Navier–Stokes Equations Try to Do?

The Navier–Stokes equations try to describe the movement of a fluid.

They try to answer questions like:

- How fast is the fluid moving?
- Which direction is it moving?
- Is it speeding up or slowing down?
- Is pressure pushing it?
- Is friction slowing it down?
- Is gravity pulling it?
- Is one part of the fluid affecting another part?

A simple way to imagine it is this:

Think of water as being made of millions of tiny invisible cubes. Each tiny cube of water is moving. It may move left, right, up, down, fast, or slow. The Navier–Stokes equations try to describe what happens to every tiny cube at every moment.

That is what makes the equations powerful, but also very difficult.

3. The Main Idea Behind the Equations

The Navier–Stokes equations are based on a simple physical idea:

Force changes motion.

This idea comes from Newton’s laws of motion. If something pushes or pulls an object, the object’s motion changes. Fluids follow the same idea, but instead of one solid object, we are studying millions of moving parts.

For a fluid, the main forces include:

Pressure: pushes fluid from high pressure to low pressure.

Viscosity: acts like internal friction and resists motion.

Gravity: pulls the fluid downward.

Movement of nearby fluid: one part of the fluid drags or affects another part.

In very simple words, the Navier–Stokes equations say:

The way a fluid changes depends on pressure, friction, outside forces, and how the fluid is already moving.

4. A Simple Version of the Equation

A simplified version of the Navier–Stokes equation looks like this:

Change in velocity + movement effect = pressure force + viscosity force + outside force

This is not the full advanced equation, but it explains the idea.

Each part has a meaning:

Change in velocity means the fluid may speed up or slow down.

Movement effect means the fluid carries its own motion with it. For example, fast-moving water can drag nearby water along.

Pressure force means the fluid is pushed from areas of high pressure to areas of low pressure.

Viscosity force means friction inside the fluid slows it down or smooths it out.

Outside force means forces like gravity or wind may affect the fluid.

NASA explains that the equations relate the motion of a fluid to quantities such as velocity, pressure, temperature, and density.

5. What Is Viscosity?

Viscosity means how thick or sticky a fluid is.

Water has low viscosity. It flows easily.

Honey has high viscosity. It flows slowly.

Air has very low viscosity, but it still has some friction. This is why air resistance exists.

Viscosity is important because it helps smooth out fluid motion. It is like a calming force.

Without viscosity, fluids would behave even more wildly.

6. What Is Turbulence?

Turbulence is messy, chaotic fluid motion.

Examples of turbulence include:

- Smoke twisting in the air
- Water splashing in a river
- Air shaking an airplane
- Waves crashing in the sea
- Wind swirling during a storm

Turbulence is one reason the Navier–Stokes equations are so difficult. Smooth flow is easier to predict. Turbulent flow is much harder because small changes can grow into big changes.

Methodology

This paper uses a simple explanatory and secondary research approach. It does not try to prove the Navier–Stokes problem because no accepted proof is currently known.

The approach has four parts.

First, the paper explains what fluids are and gives everyday examples such as water, air, smoke, and coffee.

Second, it explains the basic meaning of the Navier–Stokes equations without using advanced mathematics.

Third, it studies why fluids are hard to predict, especially when they become turbulent.

Fourth, it explains the Millennium Prize mystery: whether the equations always give smooth solutions or whether they can break down.

This method is useful for a high school research paper because the actual Navier–Stokes equations are very advanced. The goal is not to solve the problem, but to understand the mystery in simple language.

Calculations

1. A Simple Observation: Milk in Coffee

Let us imagine pouring milk into coffee.

At first, the milk moves in one direction. But very quickly, it starts to swirl. Some parts move fast. Some parts move slowly. Some parts mix into the coffee. Some parts form curved shapes.

This simple example shows that fluid motion is not always straight or predictable.

The fluid is affected by:

Factor	What It Means
---------------	----------------------

Speed	How fast the milk moves
-------	-------------------------

Direction	Where the milk is going
-----------	-------------------------

Pressure	How the liquid pushes around
----------	------------------------------

Viscosity	How much the liquid resists motion
-----------	------------------------------------

Mixing	How milk and coffee affect each other
--------	---------------------------------------

This is exactly the kind of motion the Navier–Stokes equations try to describe.

2. A Simple Observation: Smoke in Air

Smoke gives another example.

When smoke first rises from a candle, it may look smooth. But after a short time, it begins to twist and curl.

Why?

Because the smoke is affected by warm air, cooler air, gravity, pressure, and small movements in the room. Even a tiny air current can change the shape of the smoke.

This shows an important idea:

Small changes in fluids can create big effects.

That is one reason fluid motion is so difficult to predict.

3. Smooth Flow vs. Turbulent Flow

Fluids can move in different ways.

Type of Flow	Simple Meaning	Example
Smooth flow	Fluid moves in an orderly way	Slow water in a calm pipe
Turbulent flow	Fluid moves in a messy way	River water around rocks
Swirling flow	Fluid rotates or curls	Whirlpool
Mixing flow	Two fluids combine	Milk in coffee

Smooth flow is easier to study. Turbulent flow is harder because it changes quickly and looks chaotic.

4. What Does “Smooth Solution” Mean?

In mathematics, a smooth solution means the answer behaves nicely.

For the Navier–Stokes equations, a smooth solution would mean:

- The fluid speed does not become infinite.
- The pressure does not become infinite.
- The motion does not suddenly break.
- The equations keep making sense.

In simple words:

A smooth solution means the math stays under control.

5. What Does “Blow Up” Mean?

A “blow-up” does not mean a real explosion like a bomb. It means the math becomes too extreme.

For example, imagine the equations predict that the speed of a tiny part of the fluid becomes infinite. That would not be a normal answer. It would mean the equation has broken down at that point.

So the big mystery is:

Can the Navier–Stokes equations ever produce an answer that becomes infinite or not smooth?

No one knows for sure in the most important three-dimensional case. The Clay Mathematics Institute says that while these equations govern water and air, basic questions about the existence and uniqueness of solutions have not been proved.

6. Why Computers Are Not Enough

Scientists use powerful computers to simulate fluid motion. These simulations help design airplanes, predict weather, study oceans, and model blood flow.

But a computer simulation is not the same as a proof.

A computer can test many cases, but mathematics asks for certainty. To solve the Millennium Prize problem, someone must prove that the equations always work smoothly, or show one example where they fail.

That is much harder than running a simulation.

Results

1. The Equations Describe Fluid Motion

The Navier–Stokes equations describe how fluids such as water and air move. They connect velocity, pressure, viscosity, density, and outside forces. NASA describes them as equations that relate important features of moving fluids such as velocity, pressure, temperature, and density.

2. Fluids Are Hard to Predict

Fluids are difficult because they do not move as one simple object. Different parts of a fluid can move at different speeds and in different directions. One part can drag another part, spin around, or crash into nearby parts.

3. Turbulence Makes the Problem Much Harder

Turbulence is one of the biggest reasons these equations are difficult. When flow becomes messy and chaotic, it becomes much harder to predict exactly what will happen.

4. The Equations Are Useful in Real Life

The Navier–Stokes equations are used in many fields, including weather forecasting, aircraft design, engineering, medicine, ocean studies, and computer graphics.

5. The Main Mathematical Mystery Is Still Unsolved

The major unsolved problem is whether the equations always have smooth solutions in three dimensions. The Clay Mathematics Institute selected this as one of its Millennium Prize Problems, which means a correct proof could win \$1 million.

6. Simple Rules Can Lead to Huge Problems

Like the Collatz Conjecture, the Navier–Stokes problem shows that a simple-looking question can become extremely difficult. The basic idea is easy to understand: how do fluids move? But proving the answer mathematically is extremely hard.

Discussion

The Navier–Stokes equations are powerful because they connect mathematics to the real world. They are not just abstract symbols on a page. They describe things we see every day.

When wind blows through trees, that is fluid motion.

When water flows from a tap, that is fluid motion.

When clouds change shape, that is fluid motion.

When blood moves through the body, that is fluid motion.

This makes the equations very important.

But they are also very difficult. The problem is that fluids can behave in a calm way one moment and a wild way the next. A calm river can become rough when it hits rocks. Smooth smoke can suddenly turn into curls. Air around an airplane can become turbulent.

The equations try to track all of this. They must describe every point in the fluid, at every moment, while also accounting for pressure, speed, viscosity, and force. This is why the math becomes so complicated.

The main mystery is not whether the equations are useful. They are already useful. Scientists and engineers use them all the time. The mystery is whether the equations are always mathematically safe.

In other words:

Do the equations always give a proper answer, or can the answer become impossible?

This is a deep question. If the answer always stays smooth, then mathematicians need to prove it. If the answer can break down, then someone must show exactly how.

This matters because proof gives certainty. A computer simulation can show what probably happens in many cases, but proof explains what must happen in all cases. That is why the Clay Mathematics Institute says proof gives not only certainty but also understanding.

For high school students, this problem teaches an important lesson: mathematics is not only about solving textbook questions. It is also about exploring mysteries that even experts do not fully understand.

Conclusion

The Navier–Stokes equations are among the most important equations in science. They describe how fluids move, including water, air, smoke, oil, blood, and steam. They help explain everyday events like milk mixing into coffee, smoke rising in air, water flowing in rivers, and wind moving through the atmosphere.

The equations are based on simple ideas: fluids move because of speed, pressure, friction, gravity, and other forces. However, when these ideas are written mathematically, they become extremely difficult.

The biggest unsolved mystery is whether the Navier–Stokes equations always give smooth solutions in three dimensions. A smooth solution means the math stays controlled and does not become infinite or break down. No one has proved this yet, and no one has found a clear example where the equations fail.

This is why the problem is one of the Clay Millennium Prize Problems and carries a \$1 million prize. It is a perfect example of how something ordinary, like flowing water, can hide one of the deepest mysteries in mathematics.

For a high school student, the Navier–Stokes equations show that math is not just about numbers. It is also about understanding nature. Every river, cloud, storm, and swirl of smoke is part of the same beautiful mystery.

Limitations

This paper has some limitations.

First, it explains the Navier–Stokes equations in very simple language, so it does not include the full advanced mathematics.

Second, the paper does not provide a proof of the Navier–Stokes Millennium Prize Problem because no accepted proof currently exists.

Third, the examples used, such as milk in coffee and smoke in air, are simple illustrations. They help explain the idea but do not fully capture the complexity of real fluid motion.

Fourth, the actual equations involve advanced topics such as partial differential equations, vector calculus, pressure fields, and three-dimensional analysis. These are beyond the level of this paper.

Finally, computer simulations are useful, but they are not the same as mathematical proof. A full solution would require a rigorous argument accepted by expert mathematicians.

References

Clay Mathematics Institute. (n.d.). *Navier–Stokes Equation*.

Clay Mathematics Institute. (n.d.). *The Millennium Prize Problems*.

Fefferman, C. L. (n.d.). *Existence and Smoothness of the Navier–Stokes Equation*. Clay Mathematics Institute.

NASA Glenn Research Center. (n.d.). *Navier–Stokes Equations*.

COMSOL. (2015). *What Are the Navier–Stokes Equations?*