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The Collatz Conjecture: The Simple Number Game No One Can Prove

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Abstract

The Collatz Conjecture is one of the most famous unsolved problems in mathematics. What makes it surprising is that the rules are extremely simple. A student in middle school can understand and play the game: pick any positive whole number. If it is even, divide it by 2. If it is odd, multiply it by 3 and add 1. Then repeat the same steps again and again. The conjecture says that no matter which positive whole number we start with, the sequence will eventually reach 1.

For example, if we start with 6, the sequence becomes 6 → 3 → 10 → 5 → 16 → 8 → 4 → 2 → 1. This works for small numbers, large numbers, and every number mathematicians have tested so far. However, no one has been able to prove that it works for all positive whole numbers. This is why the Collatz Conjecture is so interesting: it looks easy, but it is extremely difficult.

This paper explains the Collatz Conjecture in simple language. It discusses the rules of the problem, gives examples, studies small-number data, and explains why the problem is still unsolved. The aim is to show that mathematics is not only about complicated formulas. Sometimes, the deepest mysteries begin with the simplest questions.

Introduction

Imagine a math game where the rules are so easy that almost anyone can play. You do not need algebra, geometry, calculus, or advanced formulas. You only need to know whether a number is even or odd.

The game works like this:

If the number is **even**, divide it by 2.
If the number is **odd**, multiply it by 3 and add 1.
Then keep repeating the rule.

The big question is:

Will every number eventually reach 1?

This is the Collatz Conjecture. It is also called the **3n + 1 problem** because when a number is odd, we multiply it by 3 and add 1. It is also called the **hailstone problem** because the numbers go up and down like hailstones in a storm before finally falling down to 1. Wolfram MathWorld describes the problem as one posed by Lothar Collatz in 1937, where the rule asks whether repeated steps always return to 1 for every positive starting number.

What makes this problem strange is not the rule. The rule is simple. The strange part is that nobody has proved it.

Mathematicians have tested huge numbers using computers. Every tested number eventually reaches 1. But in mathematics, testing many examples is not enough. To prove the conjecture, mathematicians must show that **every positive whole number**, forever, will reach 1.

That is what makes the Collatz Conjecture feel like a number maze. Every path we have checked seems to lead to 1, but we still do not know if every possible path does.

Research Question

Does every positive whole number eventually reach 1 when we repeatedly apply the Collatz rules?

The aim of this paper is to explain the Collatz Conjecture in a simple way so that even someone who does not enjoy mathematics can understand why it is interesting.

This paper will try to answer the following questions:

1. What is the Collatz Conjecture?
2. What are the rules of the 3n + 1 problem?
3. Why do the numbers go up and down before reaching 1?
4. What happens when we test small numbers?
5. Why is testing examples not the same as proving the conjecture?
6. Why is this problem still unsolved?
7. What does this problem teach us about mathematics?

Mathematical Theory

1. Even and Odd Numbers

The Collatz Conjecture depends on the difference between even and odd numbers.

An **even number** can be divided by 2 exactly.

Examples:

2, 4, 6, 8, 10, 12

An **odd number** cannot be divided by 2 exactly.

Examples:

1, 3, 5, 7, 9, 11

The Collatz rule treats even and odd numbers differently.

If the number is even:

n → n ÷ 2

If the number is odd:

n → 3n + 1

Here, **n** simply means the number we are using.

2. The Collatz Rule

The full rule is:

- Pick any positive whole number.
- If it is even, divide it by 2.
- If it is odd, multiply it by 3 and add 1.
- Repeat the process.

In mathematical form, the rule can be written as:

If n is even: n/2
If n is odd: 3n + 1

This rule creates a sequence, which means a list of numbers.

For example, if we start with 6:

6 → 3 → 10 → 5 → 16 → 8 → 4 → 2 → 1

After reaching 1, the numbers enter a loop:

1 → 4 → 2 → 1

So once the sequence reaches 1, it keeps repeating 1, 4, and 2 forever.

3. Why Is It Called Hailstone Numbers?

The sequences are sometimes called **hailstone sequences**. This is because the numbers often rise and fall before finally reaching 1.

For example, start with 7:

7 → 22 → 11 → 34 → 17 → 52 → 26 → 13 → 40 → 20 → 10 → 5 → 16 → 8 → 4 → 2 → 1

The number starts at 7, jumps up to 22, falls to 11, jumps up to 34, falls again, rises to 52, and then slowly comes down.

This movement looks like hailstones rising and falling inside a cloud before falling to the ground.

4. Why Is the Problem Hard?

At first, the Collatz Conjecture looks easy because the rule is easy. But the numbers can behave wildly.

Some starting numbers reach 1 quickly. Others take many steps. Some numbers climb very high before they fall. The problem is not that mathematicians cannot calculate examples. The problem is that they cannot prove what happens for **all possible numbers**.

For example, we can test 6, 7, 11, 19, and many more. But there are infinitely many positive whole numbers. No computer can check infinity.

A proof must work for every number, even numbers too large for any computer to test.

Terence Tao, one of the world's leading mathematicians, wrote that proving the Collatz Conjecture for all positive integers remains beyond current techniques.

Methodology

This paper uses a simple explanatory and data-based approach. It does not try to prove the Collatz Conjecture because no accepted proof currently exists. Instead, it explains the problem in a way that is suitable for a high school student.

The approach has four parts.

First, the paper explains the rule of the Collatz Conjecture using simple examples.

Second, it tests small starting numbers from 1 to 20 and records how many steps each number takes to reach 1.

Third, it compares the behaviour of different starting numbers to see whether some numbers take longer than others.

Fourth, it discusses why this data supports the conjecture but does not prove it.

This method is useful because the Collatz Conjecture is easy to experiment with. Even without advanced mathematics, a student can explore the problem, collect data, and notice patterns.

Data Analysis

1. Example Starting with 6

Let us begin with 6.

6 is even, so divide by 2:

$6 \div 2 = 3$

3 is odd, so multiply by 3 and add 1:

$3 \times 3 + 1 = 10$

10 is even:

$10 \div 2 = 5$

5 is odd:

$5 \times 3 + 1 = 16$

16 is even:

$16 \div 2 = 8$

Then:

$8 \rightarrow 4 \rightarrow 2 \rightarrow 1$

So the full sequence is:

$6 \rightarrow 3 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$

This takes **8 steps** to reach 1.

2. Example Starting with 11

Now let us try 11.

11 is odd:

3 × 11 + 1 = 34

34 is even:

34 ÷ 2 = 17

17 is odd:

3 × 17 + 1 = 52

Then the sequence continues:

11 → 34 → 17 → 52 → 26 → 13 → 40 → 20 → 10 → 5 → 16 → 8 → 4 → 2 → 1

This takes **14 steps** to reach 1.

This example shows that a number can go up before it comes down.

3. Example Starting with 7

Start with 7:

7 → 22 → 11 → 34 → 17 → 52 → 26 → 13 → 40 → 20 → 10 → 5 → 16 → 8 → 4 → 2 → 1

This takes **16 steps**.

The highest number reached is **52**.

This is interesting because we started with only 7, but the sequence climbed all the way to 52 before falling to 1.

4. Testing Numbers from 1 to 20

The table below shows how many steps each number from 1 to 20 takes to reach 1.

Starting Number	Steps to Reach 1	Highest Number Reached
1	0	1
2	1	2
3	7	16
4	2	4
5	5	16

Starting Number	Steps to Reach 1	Highest Number Reached
6	8	16
7	16	52
8	3	8
9	19	52
10	6	16
11	14	52
12	9	16
13	9	40
14	17	52
15	17	160
16	4	16
17	12	52
18	20	52
19	20	88
20	7	20

5. What the Data Shows

The data shows that every number from 1 to 20 reaches 1.

However, they do not all take the same number of steps.

For example:

2 reaches 1 in only 1 step.
6 reaches 1 in 8 steps.
7 reaches 1 in 16 steps.
18 and **19** take 20 steps.

This means that even small numbers can behave very differently.

The data also shows that some numbers rise much higher before falling. For example, starting with 15 reaches a highest value of 160 before coming down to 1.

This is why the problem is difficult. A number may look small at the beginning, but its path can suddenly become large and unpredictable.

6. Why This Is Not a Proof

The table gives evidence that the Collatz rule works for the numbers 1 to 20. But it does not prove the conjecture for all numbers.

To understand why, imagine checking the first million numbers. Even if all of them reach 1, there could still be a larger number that does not. Then imagine checking the first billion numbers. The same problem remains.

A mathematical proof must not depend only on examples. It must explain why the rule always works for every positive whole number.

That is the missing piece.

Results

1. The Rule Is Simple

The Collatz Conjecture uses only two basic operations: dividing by 2 and multiplying by 3 and adding 1. This makes it easy for anyone to understand and test.

2. Every Tested Small Number Reaches 1

In the sample from 1 to 20, every number eventually reaches 1. This supports the conjecture, but it does not prove it.

3. Some Numbers Take Much Longer Than Others

Different starting numbers behave differently. Some reach 1 quickly, while others take many more steps.

For example, 8 reaches 1 in only 3 steps, but 18 and 19 take 20 steps.

4. Numbers Can Rise Before Falling

The sequence does not always go downward. Some numbers first climb much higher than the starting number.

For example, 15 rises to 160 before eventually reaching 1.

5. The Problem Remains Unsolved

No one has found a positive whole number that does not reach 1. However, no one has proved that all positive whole numbers must reach 1.

This is why the Collatz Conjecture remains an open problem in mathematics.

6. Advanced Work Has Been Done, but the Full Mystery Remains

Mathematicians have made partial progress. For example, Terence Tao proved an important result showing that “almost all” Collatz orbits come close to bounded values in a technical mathematical sense, but this is still not a complete proof of the conjecture.

Discussion

The Collatz Conjecture is fascinating because it breaks one of our expectations about mathematics. Many people think difficult math problems must have difficult rules. But Collatz shows that a problem can have simple rules and still be extremely hard.

The rule feels like a game. If the number is even, cut it in half. If the number is odd, make it bigger by doing $3n + 1$. This creates a push-and-pull effect. Even numbers shrink. Odd numbers grow. But after an odd number is multiplied by 3 and 1 is added, the result is always even. That means the sequence can usually be divided by 2 soon after.

For example:

If $n = 5$:

$$\mathbf{3 \times 5 + 1 = 16}$$

Then:

$$\mathbf{16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1}$$

This looks like the sequence is being pulled downward. But this does not always happen quickly. Some numbers rise again and again before falling.

This is where the mystery begins.

One way to think about it is like climbing a mountain in fog. Sometimes the path goes down. Sometimes it suddenly goes up. You may believe the path will eventually lead to the ground, but you cannot see the entire mountain. That is what mathematicians face with Collatz. Every tested path seems to come down to 1, but no one can see all possible paths at once.

Another way to understand the problem is to compare it to a maze. Every number is an entrance. The conjecture says every entrance leads to the same exit: 1. So far, every entrance that has been tested does lead to 1. But since there are infinitely many entrances, mathematicians need a proof that covers the whole maze.

The Collatz Conjecture also shows the difference between **evidence** and **proof**.

Evidence means we have many examples that support an idea. Proof means we know with complete certainty that the idea is true in every case.

For science, evidence is often enough to build strong theories. But in mathematics, proof is the gold standard. A statement is not considered solved until there is a complete logical argument.

This is why the Collatz Conjecture has survived for so long. It is not because people have ignored it. Many mathematicians have studied it. Computers have tested it. But the final proof has not been found.

Conclusion

The Collatz Conjecture is one of the most interesting problems in number theory because it is simple to understand but difficult to prove. The rule is easy: if a number is even, divide it by 2; if it is odd, multiply it by 3 and add 1. The mystery is whether every positive whole number eventually reaches 1.

In this paper, we tested numbers from 1 to 20. Every number reached 1, but the number of steps varied. Some numbers reached 1 quickly, while others took longer. Some numbers also climbed much higher before falling back down.

These examples support the Collatz Conjecture, but they do not prove it. The reason is that mathematics requires proof for all possible positive whole numbers, not just tested examples. Since there are infinitely many numbers, checking examples can never be enough.

The Collatz Conjecture teaches an important lesson: simple mathematics can lead to deep mysteries. It shows that math is not only about formulas in textbooks. It is also about curiosity, patterns, puzzles, and unanswered questions.

Even today, the Collatz Conjecture remains unsolved. It continues to attract students, teachers, computer scientists, and mathematicians because anyone can understand the question, but no one has fully solved it.

Limitations

This paper has some limitations.

First, it explains the Collatz Conjecture in simple language, so it does not include advanced mathematical theories in detail.

Second, the paper does not provide a proof of the conjecture because no accepted proof currently exists.

Third, the data analysis only tests numbers from 1 to 20. This is useful for understanding the pattern, but it is not enough to prove the conjecture.

Fourth, computer testing is not the same as mathematical proof. Even if computers test billions or trillions of numbers, there are still infinitely many numbers left.

Finally, this paper focuses only on positive whole numbers. The Collatz rule can be studied in other number systems too, but those versions are more advanced and are not included here.

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